

Reconsidering a higher-spin-field solution to the main cosmological constant problem

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Abstract

Following an earlier suggestion by Dolgov, we present a model of two massless vector fields which dynamically cancel a cosmological constant of arbitrary magnitude and sign. Flat Minkowski spacetime appears asymptotically as an attractor of the field equations. Unlike the original model, the new model does not upset the local Newtonian gravitational dynamics.

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I. INTRODUCTION

The main cosmological constant problem [1, 2] can be phrased as follows: *why do the quantum fields of the vacuum state not naturally produce a large (positive or negative) value for the cosmological constant with an energy scale of the order of the known energy scales of elementary particle physics?*

An ideal solution would be to compensate dynamically any cosmological constant there may be. In equilibrium, such a compensation appears to be impossible with a constant (spacetime-independent) fundamental scalar field [1]. Partly for this reason, Dolgov [3, 4] has proposed using nonconstant higher-spin fields, notably a nonconstant vector field. He presented a remarkably simple cosmological model with a single massless vector field $A_\alpha(x)$, which allows for the compensation of a cosmological constant Λ of a particular sign with Minkowski spacetime appearing asymptotically as an attractor of the dynamical field equations. However, a serious flaw of this compensation-type solution to the cosmological constant problem was pointed out by Rubakov and Tinyakov [5], namely, that the resulting Minkowski spacetime (with a vector-field background canceling Λ) implies an unacceptable modification of the standard Newtonian gravitational dynamics for small systems.

In this article, we present a specific model with two massless vector fields, $A_\alpha(x)$ and $B_\beta(x)$, which evades the above-mentioned flaw with the local Newtonian dynamics. Inspiration for this model was obtained from previous work by Volovik and one of the present authors on the so-called q -theory approach [6–9] to the main cosmological constant problem (a one-page review of q -theory can be found in Appendix A of Ref. [10] and a ultrabrief summary will be given in the Footnote of Sec. II A). In Ref. [9], in particular, it was realized that the Dolgov theory actually provides a generalization of q -theory, with the genuine q -theory appearing asymptotically. Therefore, the insights of q -theory can also be applied to Dolgov-type vector-field models and be used to overcome the Newtonian-dynamics flaw.

II. MINKOWSKI ATTRACTOR FROM A VECTOR FIELD

A. Generalized Dolgov model

Our starting point is the vector-field model presented by Dolgov [3, 4] (related aether-type theories have been discussed by, for example, Jacobson [11]). Here, we extend the previous analysis of Ref. [9], in order to compensate both positive and negative cosmological constants in a single model.

The effective action of the massless vector field $A_\alpha(x)$ and the metric $g_{\alpha\beta}(x)$ is taken to

be the following ($\hbar = c = 1$):

$$S_{\text{eff}}[A, g] = - \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G_N} R[g] + \epsilon(Q[A, g]) + \Lambda \right), \quad (2.1a)$$

$$Q[A, g] \equiv \sqrt{A_{\alpha;\beta} A^{\alpha;\beta}}, \quad (2.1b)$$

where ϵ is an appropriate function of the variable Q (semicolons in the definition of Q denote covariant differentiation), R is the Ricci scalar, G_N is Newton's gravitational constant, and Λ is the effective cosmological constant. The above action generalizes the one of Dolgov [4] which has $\epsilon = -\eta_0 Q^2$ for $\eta_0 = \pm 1$. As mentioned in Ref. [4], the consistency of such a massless vector field A_α at the quantum level (e.g., unitarity) needs to be investigated, but this issue lies outside the scope of the present paper which is primarily concerned with the classical dynamics of the metric and vector fields. Incidentally, this vector field A_α is not a gauge field, so also its masslessness needs to be explained, but, again, this issue will not be addressed here.

It may be useful to present a simple example of a bounded function $\epsilon(Q)$, which gives a unique equilibrium value Q_0 for each value of the cosmological constant Λ :

$$\epsilon(Q) = \begin{cases} \epsilon_{\text{max}} \left(1 - \sqrt{1 - (Q - Q_m)^2 / (\Delta Q)^2} \right), & \text{for } Q \in (Q_m - \Delta Q, Q_m + \Delta Q), \\ 0, & \text{otherwise,} \end{cases} \quad (2.2)$$

for $Q_m > \Delta Q > 0$ and a constant $\epsilon_{\text{max}} > 0$. The corresponding gravitating vacuum energy density, given by $\tilde{\epsilon}(Q) \equiv \epsilon(Q) - Q d\epsilon(Q)/dQ$ according to Ref. [6], descends monotonically from $+\infty$ to $-\infty$ as Q runs from $Q_m - \Delta Q$ to $Q_m + \Delta Q$. This behavior of $\tilde{\epsilon}(Q)$ indeed allows for the compensation of any value of Λ by a unique equilibrium value Q_0 [see, in particular, (2.4b) below].

As suggested by Dolgov [3, 4], the following isotropic *Ansatz* can be taken for the vector field $A_\alpha(x)$ in a spatially flat Friedmann–Robertson–Walker (FRW) universe:

$$A_0 = A_0(t) \equiv V(t), \quad A_1 = A_2 = A_3 = 0, \quad (2.3a)$$

$$(g_{\alpha\beta}) = \text{diag}(1, -a(t), -a(t), -a(t)), \quad (2.3b)$$

where t is the cosmic time and $a(t)$ the FRW cosmic scale factor with Hubble parameter $H \equiv (da/dt)/a$.

The reduced field equations are given in App. A. With appropriate boundary conditions (consistent with the Friedmann equation), numerical solutions have been obtained. These numerical solutions show that, for either sign of the cosmological constant Λ , there exists a *finite* domain of boundary values V and dV/dt at $t = t_{\text{start}}$, which give the same asymptotic

solution for $t \rightarrow \infty$:

$$V(t) \sim (Q_0/2)t, \quad H(t) \sim 1/t. \quad (2.4a)$$

The particular value Q_0 entering the dynamical solution (2.4a) precisely cancels the effects from the cosmological constant [6],

$$\Lambda + \left[\epsilon(Q) - Q \frac{d\epsilon(Q)}{dQ} \right]_{Q=Q_0} \equiv \Lambda + \tilde{\epsilon}(Q_0) = 0, \quad (2.4b)$$

as the fundamental dynamic variable $A^\alpha_\beta \equiv A^\alpha_{;\beta} \equiv \nabla_\beta A^\alpha$ approaches the Lorentz-invariant tensor structure [9] characteristic of q -theory,¹

$$A^\alpha_\beta(x) \Big|_{\text{equil}} = \frac{1}{2} Q_0 \delta^\alpha_\beta. \quad (2.4c)$$

This shows that Minkowski spacetime can appear asymptotically as an *attractor* of the dynamical equations considered, independent of the sign of the cosmological constant (figures similar to Fig. 2 of Ref. [9] have been obtained but will not be given here).

B. Flawed Newtonian dynamics

Rubakov and Tinyakov [5] considered the quadratic action of small changes in the fields away from the attractor solution (2.4a):

$$A_\alpha(x) = A^\text{sol}_\alpha(x) + \hat{v}_\alpha(x) \sim (t Q_0/2) \delta^0_\alpha + \hat{v}_\alpha(x), \quad (2.5a)$$

$$g_{\alpha\beta}(x) = g^\text{sol}_{\alpha\beta}(x) + \hat{h}_{\alpha\beta}(x) \sim \eta_{\alpha\beta} + \hat{h}_{\alpha\beta}(x), \quad (2.5b)$$

where the perturbed fields are distinguished by a hat in order to avoid confusion later on. From (2.1) and (2.3), they obtain the following structure of the field equation for the metric perturbation $\hat{h}_{\alpha\beta}(x)$:

$$(8\pi G_N)^{-1} \left\{ \partial^2 \hat{h} \right\}^{(\text{GR})} + (A_0)^2 \partial^2 \hat{h} = T_{\text{ext}}, \quad (2.6)$$

¹ Very briefly, q -theory aims to give the proper macroscopic description of the Lorentz-invariant quantum vacuum where a (Planck-scale) cosmological constant Λ has been canceled dynamically by appropriate microscopic degrees of freedom (see the original article [6] for further details and also the brief review [10]). Typically, there are one or more of these Lorentz-invariant vacuum variables (denoted by q , with or without additional suffixes) to characterize the thermodynamics of this *static* physical system in equilibrium. A particular example of q -theory is given by the Lorentz-invariant vacuum variable Q_0 appearing in (2.4c) with a value determined by the Gibbs-Duhem-type equilibrium condition (2.4b). The issue discussed here is the *dynamics*, namely, how the equilibrium state is approached.

where the notation is symbolic with all spacetime indices omitted. The two occurrences of “ $\partial^2 \hat{h}$ ” in (2.6) stand for different expressions, each involving two partial derivatives ∂_α , the Minkowski metric $\eta_{\alpha\beta}$, and the metric-field components $\hat{h}_{\alpha\beta}$. On the right-hand side of (2.6) appears the energy-momentum tensor $T_{\text{ext}}^{\alpha\beta}$ of a local matter distribution. Note that (2.6) for $A_0 \equiv 0$ corresponds to the standard Einstein equation of general relativity (GR), which reproduces the Poisson equation of Newtonian gravity in the nonrelativistic limit.

With $A_0 \sim (Q_0/2)t$, $Q_0 \sim (8\pi G_N)^{-1} \sim (10^{18} \text{ GeV})^2$, and $t \sim 10^{10} \text{ yr} \sim (10^{-33} \text{ eV})^{-1}$, the second term on the left-hand side of (2.6) dominates the first term and ruins the standard Newtonian behavior. This equation also suggests that the properties of gravitational waves are unusual compared to those from general relativity and, most likely, physically unacceptable [5].

III. MINKOWSKI ATTRACTOR FROM TWO VECTOR FIELDS

A. Setup

A possible cure for the flaw of Sec. II B uses two massless vector fields $A_\alpha(x)$ and $B_\alpha(x)$ with the following effective action:

$$S_{\text{eff}} = - \int d^4x \sqrt{-g} \left(\frac{1}{2} (E_{\text{Planck}})^2 R + \epsilon(Q_A, Q_B) + \Lambda \right), \quad (3.1a)$$

$$Q_A \equiv \sqrt{A_{\alpha;\beta} A^{\alpha;\beta}}, \quad Q_B \equiv \sqrt{B_{\alpha;\beta} B^{\alpha;\beta}}, \quad (3.1b)$$

$$E_{\text{Planck}} \equiv (8\pi G_N)^{-1/2} \approx 2.44 \times 10^{18} \text{ GeV}. \quad (3.1c)$$

The Dolgov-type *Ansatz* for the vector fields $A_\alpha(x)$ and $B_\beta(x)$ and for the metric $g_{\alpha\beta}(x)$ is:

$$A_0 = A_0(t) \equiv V(t), \quad A_1 = A_2 = A_3 = 0, \quad (3.2a)$$

$$B_0 = B_0(t) \equiv W(t), \quad B_1 = B_2 = B_3 = 0, \quad (3.2b)$$

$$(g_{\alpha\beta}) = \text{diag}(1, -a(t), -a(t), -a(t)), \quad (3.2c)$$

where $a(t)$ is again the cosmic scale factor of the spatially flat FRW universe considered. Solving the field equations from (3.1a) for the *Ansatz* fields (3.2) gives the explicit functions $\overline{V}(t)$, $\overline{W}(t)$, and $\overline{a}(t)$.

For later use, we introduce dimensionless variables. Specifically, we replace the above dimensional variables (and the variable X to be defined shortly) by the following dimensionless

variables:

$$\{\Lambda, \epsilon, X, t, H\} \rightarrow \{\lambda, e, \chi, \tau, h\}, \quad (3.3a)$$

$$\{Q_A, Q_B, V, W\} \rightarrow \{q_A, q_B, v, w\}, \quad (3.3b)$$

having used appropriate powers of the reduced Planck energy E_{Planck} without additional numerical factors. Moreover, $|\lambda|$ is considered to be of order unity.

B. Main argument

The action-density term $\epsilon(Q_A, Q_B)$ for the two vector fields of the model (3.1a) will be designed to cancel the effects of the cosmological constant Λ and to give a vanishing contribution to the field equation for the metric perturbation $\hat{h}_{\alpha\beta}(x)$. Concretely, perturbations around the background solution from (3.1a) and (3.2) give the following equation instead of (2.6):

$$(8\pi G_N)^{-1} \left\{ \partial^2 \hat{h} \right\}^{(\text{GR})} + [X^{-1}]_{\text{asympt}} \left\{ t^2 \partial^2 \hat{h} + t \partial \hat{h} + \hat{h} \right\} + [\epsilon - \tilde{\epsilon}]_{\text{asympt}} \left\{ t^2 \partial^2 \hat{h} + t \partial \hat{h} + \hat{h} \right\} + [\Lambda + \tilde{\epsilon}]_{\text{asympt}} \left\{ \hat{h} \right\} = T_{\text{ext}}, \quad (3.4a)$$

$$[X^{-1}]_{\text{asympt}} = 0, \quad (3.4b)$$

$$[\epsilon - \tilde{\epsilon}]_{\text{asympt}} = 0, \quad (3.4c)$$

$$[\Lambda + \tilde{\epsilon}]_{\text{asympt}} = 0. \quad (3.4d)$$

Equation (3.4a) for the metric perturbation contains the asymptotic values of two basic quantities of q -theory [6, 8], namely, the inverse of the vacuum compressibility (denoted by the Greek capital letter ‘chi’),

$$X^{-1} \equiv Q_A^2 \frac{d^2 \epsilon(Q_A, Q_B)}{dQ_A dQ_A} + Q_B^2 \frac{d^2 \epsilon(Q_A, Q_B)}{dQ_B dQ_B} + 2 Q_A Q_B \frac{d^2 \epsilon(Q_A, Q_B)}{dQ_A dQ_B}, \quad (3.5a)$$

and the thermodynamically active (and gravitating) vacuum energy density,

$$\tilde{\epsilon} \equiv \epsilon - Q_A \frac{d\epsilon}{dQ_A} - Q_B \frac{d\epsilon}{dQ_B}. \quad (3.5b)$$

Physically, conditions (3.4b) and (3.4c) can be interpreted as having a perfectly soft and flexible medium (isothermal compressibility $X \equiv -V^{-1} dV/dP = \infty$), which does not affect the metric perturbations. But, for the moment, we are only interested in finding a working model and follow Newton’s advice, “*Hypotheses non fingo*” [12].

The derivation of (3.4a) proceeds in five steps. First, consider the second-order variation of the action-density term $\epsilon(Q_A, Q_B)$,

$$\begin{aligned} & \left[Q_A^2 \frac{d^2 \epsilon}{dQ_A dQ_A} \right] \frac{\delta Q_A^{(1)}}{Q_A} \frac{\delta Q_A^{(1)}}{Q_A} + \left[Q_B^2 \frac{d^2 \epsilon}{dQ_B dQ_B} \right] \frac{\delta Q_B^{(1)}}{Q_B} \frac{\delta Q_B^{(1)}}{Q_B} \\ & + \left[2 Q_A Q_B \frac{d^2 \epsilon}{dQ_A dQ_B} \right] \frac{\delta Q_A^{(1)}}{Q_A} \frac{\delta Q_B^{(1)}}{Q_B} + \left[Q_A \frac{d\epsilon}{dQ_A} \right] \frac{\delta Q_A^{(2)}}{Q_A} + \left[Q_B \frac{d\epsilon}{dQ_B} \right] \frac{\delta Q_B^{(2)}}{Q_B}, \quad (3.6) \end{aligned}$$

where all factors in square brackets are evaluated with the background solutions $\bar{V}(t)$, $\bar{W}(t)$, and $\bar{a}(t)$ from (3.1a) and (3.2).

Second, observe that all factors $\delta Q_X^{(1)}/Q_X$ in the above equation have the same structure and so do the factors $\delta Q_X^{(2)}/Q_X$. In terms of the perturbative fields \hat{v} , \hat{w} , and \hat{h} [the definitions of \hat{v}_α and $\hat{h}_{\alpha\beta}$ have been given in (2.5), the one of \hat{w}_α is similar], (3.6) becomes in a symbolic notation:

$$\begin{aligned} & \left[Q_A^2 \frac{d^2 \epsilon}{dQ_A dQ_A} \right] f(\hat{v}, \hat{h})^2 + \left[Q_B^2 \frac{d^2 \epsilon}{dQ_B dQ_B} \right] f(\hat{w}, \hat{h})^2 + \left[2 Q_A Q_B \frac{d^2 \epsilon}{dQ_A dQ_B} \right] f(\hat{v}, \hat{h}) f(\hat{w}, \hat{h}) \\ & + \left[Q_A \frac{d\epsilon}{dQ_A} \right] i(\hat{v}, \hat{h}) + \left[Q_B \frac{d\epsilon}{dQ_B} \right] i(\hat{w}, \hat{h}), \quad (3.7) \end{aligned}$$

where the explicit expressions for the linear function f and the quadratic function i can be obtained from the results given in App. B.

Third, assume certain (anti-)symmetry properties of $\epsilon(Q_A, Q_B)$ and its derivatives and also the existence of a Dolgov-type asymptotic background solution [both assumptions are satisfied by the specific example of Sec. IIIC for $\delta = 0$]. Then, it can be shown that the resulting equations for \hat{v} and \hat{w} have an identical solution, provided the matter perturbation is localized. The implication is that the first three terms in (3.7) combine and so do the last two terms. Indeed, a direct calculation gives:

$$\begin{aligned} & \left[Q_A^2 \frac{d^2 \epsilon}{dQ_A dQ_A} + Q_B^2 \frac{d^2 \epsilon}{dQ_B dQ_B} + 2 Q_A Q_B \frac{d^2 \epsilon}{dQ_A dQ_B} \right] \left\{ g_2 \hat{\partial} \hat{h} \hat{\partial} \hat{h} + g_1 \hat{h} \hat{\partial} \hat{h} + g_0 \hat{h} \hat{h} \right\} \\ & + \left[Q_A \frac{d\epsilon}{dQ_A} + Q_B \frac{d\epsilon}{dQ_B} \right] \left\{ k_2 \hat{\partial} \hat{h} \hat{\partial} \hat{h} + k_1 \hat{h} \hat{\partial} \hat{h} + k_0 \hat{h} \hat{h} \right\} \\ & = [X^{-1}] \left\{ g_2 \hat{\partial} \hat{h} \hat{\partial} \hat{h} + g_1 \hat{h} \hat{\partial} \hat{h} + g_0 \hat{h} \hat{h} \right\} + [\epsilon - \tilde{\epsilon}] \left\{ k_2 \hat{\partial} \hat{h} \hat{\partial} \hat{h} + k_1 \hat{h} \hat{\partial} \hat{h} + k_0 \hat{h} \hat{h} \right\}, \quad (3.8) \end{aligned}$$

with a symbolic notation for the prefactors g_n and k_n .

Fourth, consider the further (standard) contribution to the quadratic action, which follows from the variation of the metric entering the spacetime measure of (3.1a). Specifically, this contribution is

$$[\Lambda + \epsilon] \left\{ l_0 \hat{h} \hat{h} \right\}, \quad (3.9)$$

again with a symbolic notation for the prefactor l_0 .

Fifth, make the necessary partial integrations in (3.8), while assuming X^{-1} and $(\epsilon - \tilde{\epsilon})$ to be constant on the macroscopic length scales considered, and add the contribution (3.9). The resulting quadratic action then gives the linear field equation (3.4a). Note that, in the last term on the left-hand side of (3.4a), the asymptotic value of ϵ has been replaced by the one of $\tilde{\epsilon}$, in agreement with (3.4c).

It now remains for us to present an *Ansatz* for $\epsilon(Q_A, Q_B)$ with appropriate symmetry properties and with both $[X^{-1}]$ and $[\epsilon - \tilde{\epsilon}]$ vanishing identically (that is, purely by algebra). This will be done in the next subsection.

C. Specific model

Following up on the general discussion of Sec. IIIB, we now choose the action density $e(Q_A, Q_B)$ of (3.1a) to be a particular rational function. Specifically, the dimensionless vacuum energy density e , the corresponding gravitating vacuum energy density \tilde{e} , and the corresponding inverse vacuum compressibility χ^{-1} are given by:

$$e = \frac{(A_{\alpha;\beta} A^{\alpha;\beta})^2 - (B_{\alpha;\beta} B^{\alpha;\beta})^2}{(E_{\text{Planck}})^8 \delta + (A_{\alpha;\beta} A^{\alpha;\beta})(B_{\alpha;\beta} B^{\alpha;\beta})} = \frac{q_A^4 - q_B^4}{\delta + q_A^2 q_B^2} = \frac{q_A^4 - q_B^4}{q_A^2 q_B^2} + \mathcal{O}(\delta), \quad (3.10a)$$

$$\tilde{e} \equiv e - q_A \frac{de}{dq_A} - q_B \frac{de}{dq_B} = \frac{(q_A^2 q_B^2 - 3\delta)(q_A^4 - q_B^4)}{(\delta + q_A^2 q_B^2)^2} = \frac{q_A^4 - q_B^4}{q_A^2 q_B^2} + \mathcal{O}(\delta), \quad (3.10b)$$

$$\begin{aligned} \chi^{-1} &\equiv q_A^2 \frac{d^2 e}{dq_A dq_A} + q_B^2 \frac{d^2 e}{dq_B dq_B} + 2 q_A q_B \frac{d^2 e}{dq_A dq_B} \\ &= -4\delta \frac{(5q_A^2 q_B^2 - 3\delta)(q_A^4 - q_B^4)}{(\delta + q_A^2 q_B^2)^3} = -20\delta \frac{\tilde{e}}{q_A^2 q_B^2} + \mathcal{O}(\delta^2), \end{aligned} \quad (3.10c)$$

for a positive infinitesimal δ and dimensionless variables from (3.3). The last steps in the above three equations give the leading order in δ . It is certainly possible to set δ immediately to zero in (3.10), but we prefer to keep δ explicit in order to clarify two technical points later on, regarding asymptotes and stability. It needs, however, to be emphasized that the actual model function $\epsilon(Q_A, Q_B)$ for the action (3.1a) is the one obtained from (3.10a) with $\delta = 0$ exactly, so that Eqs. (3.4b) and (3.4c) hold identically.

From the Dolgov-type *Ansatz* (3.2), the following ordinary differential equations (ODEs)

for $v(\tau)$, $w(\tau)$, and $h(\tau)$ are obtained (cf. App. A):

$$\left[\left(\ddot{v} + 3h\dot{v} - 3h^2v \right) \frac{de}{q_A dq_A} + \dot{v} \frac{d}{d\tau} \left(\frac{de}{q_A dq_A} \right) \right]_{q_A=\sqrt{\dot{v}^2+3h^2v^2}, q_B=\sqrt{\dot{w}^2+3h^2w^2}} = 0, \quad (3.11a)$$

$$\left[\left(\ddot{w} + 3h\dot{w} - 3h^2w \right) \frac{de}{q_B dq_B} + \dot{w} \frac{d}{d\tau} \left(\frac{de}{q_B dq_B} \right) \right]_{q_A=\sqrt{\dot{v}^2+3h^2v^2}, q_B=\sqrt{\dot{w}^2+3h^2w^2}} = 0, \quad (3.11b)$$

$$2\dot{h} + 3h^2 = \lambda + \left[\tilde{e}(q_A, q_B) - \frac{d}{d\tau} \left(h v^2 \frac{de}{q_A dq_A} \right) + \dot{v}^2 \frac{de}{q_A dq_A} - \frac{d}{d\tau} \left(h w^2 \frac{de}{q_B dq_B} \right) + \dot{w}^2 \frac{de}{q_B dq_B} \right]_{q_A=\sqrt{\dot{v}^2+3h^2v^2}, q_B=\sqrt{\dot{w}^2+3h^2w^2}}, \quad (3.11c)$$

where an overdot stands for differentiation with respect to τ . The corresponding Friedmann equation is given by

$$3h^2 = \lambda + \left[\tilde{e}(q_A, q_B) \right]_{q_A=\sqrt{\dot{v}^2+3h^2v^2}, q_B=\sqrt{\dot{w}^2+3h^2w^2}}. \quad (3.12)$$

The boundary conditions for $v(\tau)$, $\dot{v}(\tau)$, $w(\tau)$, $\dot{w}(\tau)$, and $h(\tau)$ at $\tau = \tau_{\text{start}}$ must satisfy (3.12) with a nonnegative right-hand side. Observe also that, just as in (2.4b) for the generalized Dolgov model of Sec. II A, the right-hand side of (3.12) can be nullified by an asymptotic solution with the appropriate constant value for the ratio of the auxiliary variables q_A and q_B .

The asymptotic behavior of the solutions of (3.11) is rather subtle. Mathematically, the order of limits $\delta \downarrow 0$ and $\tau \rightarrow \infty$ is important. Physically, we take a fixed extremely small value of δ and consider only “modest” values of the dimensionless cosmic time τ :

$$\delta = 10^{-10^{10}}, \quad \tau \leq 10^{60}, \quad (3.13)$$

where the last inequality includes cosmic times up to the present age of the Universe in units of $t_{\text{Planck}} \equiv 1/E_{\text{Planck}}$. It is, of course, possible to take a less radical value for δ , but the one in (3.13) dispenses with some unnecessary discussion later on.

For appropriate boundary conditions at $\tau = \tau_{\text{start}} = \mathcal{O}(1)$ and small but finite values of δ , the solutions of (3.11) have the following asymptotic behavior for $\tau \rightarrow \infty$:

$$v \sim k \tau^p, \quad w \sim l \tau^p, \quad h \sim n \tau^{-1}, \quad (3.14a)$$

$$k^2/l^2 = \sqrt{(\lambda/2)^2 + 1} - \lambda/2, \quad (3.14b)$$

$$0 = p(p-1) - 3np + 3n^2, \quad (3.14c)$$

$$0 = \delta [p^2 - 16np + 5n(4+3n)], \quad (3.14d)$$

where the parameter ratio (3.14b) follows from (3.12), the relation (3.14c) from (3.11a), and the relation (3.14d) valid for $p > 1$ from the pressure terms in (3.11c). Equations

(3.14c) and (3.14d) for $\delta \neq 0$ give two sets of values (n, p) with $p > 1$. One set has values $(\tilde{n}, \tilde{p}) \approx (0.6480, 2.424)$. But it is the other set, with values

$$\bar{n} = \frac{2}{183} \left[83 + \sqrt{14441} \cos \left(\frac{1}{3} \arccos \frac{973771}{(14441)^{3/2}} \right) \right] \approx 2.152, \quad (3.15a)$$

$$\bar{p} = 4\bar{n} \frac{3\bar{n} + 5}{13\bar{n} - 1} \approx 3.655, \quad (3.15b)$$

which will turn out to be relevant for the numerical results to be presented shortly. As mentioned in Sec. IIIB, the Dolgov-type asymptotic solution (3.14a) enters the derivation of (3.4a), as do the (anti-)symmetry properties of (3.10a) and its derivatives for $\delta = 0$.

D. Numerical solutions

For $\lambda = \pm 2$, $\delta = 10^{-10}$, and boundary conditions in an appropriate domain, the numerical solutions of the reduced fields equations display an attractor-type behavior (Fig. 1) with $v \propto \tau^{\bar{p}}$, $w \propto \tau^{\bar{p}}$, and $h \sim \bar{n} \tau^{-1}$ for coefficients \bar{p} and \bar{n} from (3.15). Different λ values and different boundary conditions are seen to give an identical asymptote $h \sim \bar{n} \tau^{-1}$ [remark that the normalization of $v(\tau)$ is irrelevant for this asymptote; what matters is the constant ratio q_A/q_B]. Within the numerical accuracy, the same results have been obtained for $\delta = 0$. The issue of the allowed boundary conditions is, however, more complicated and a complete analysis is left for a future publication. Another topic for future investigations is the possible cusp-like behavior at $\tau \sim 1.6$ suggested by two $\lambda = -2$ solutions $h(\tau)$ in Fig. 1.

The h panels in the third column of Fig. 1 show the main result: the approximately constant Hubble parameter h of a de-Sitter-like universe at $\tau \sim 1$ changes to $h \sim \tau^{-1}$ for $\tau \gg 1$, so that a Minkowski spacetime (with $h = 0$) is approached asymptotically. Moreover, the flat spacetime obtained for small but nonzero δ has inverse vacuum compressibility $\chi_{\text{asyp}}^{-1} = 0$ as $\bar{p} > 1$. The actual \bar{p} value from (3.15) even gives $\lim_{\tau \rightarrow \infty} \tau^2 \chi^{-1}(\tau) = 0$, as required by (3.4a). Of course, $[\chi^{-1}(\tau)]$ vanishes identically for $\delta = 0$, and so does the quantity $[e(\tau) - \tilde{e}(\tau)]$. Finally, the total gravitating vacuum energy density $[\lambda + \tilde{e}(\tau)]$ is found to drop to zero as τ^{-2} , in agreement with (3.12). As remarked already in Eq. (5.16) of Ref. [7] (and reiterated in the review [10]), the present value of this quantity $[\Lambda + \tilde{e}(t_{\text{now}})]$ would be of the order of the experimental value $\Lambda^{(\text{exp})} \sim (\text{meV})^4$.

E. Remarks

Several points about the proposed Λ -cancellation mechanism of this section are to be noted:

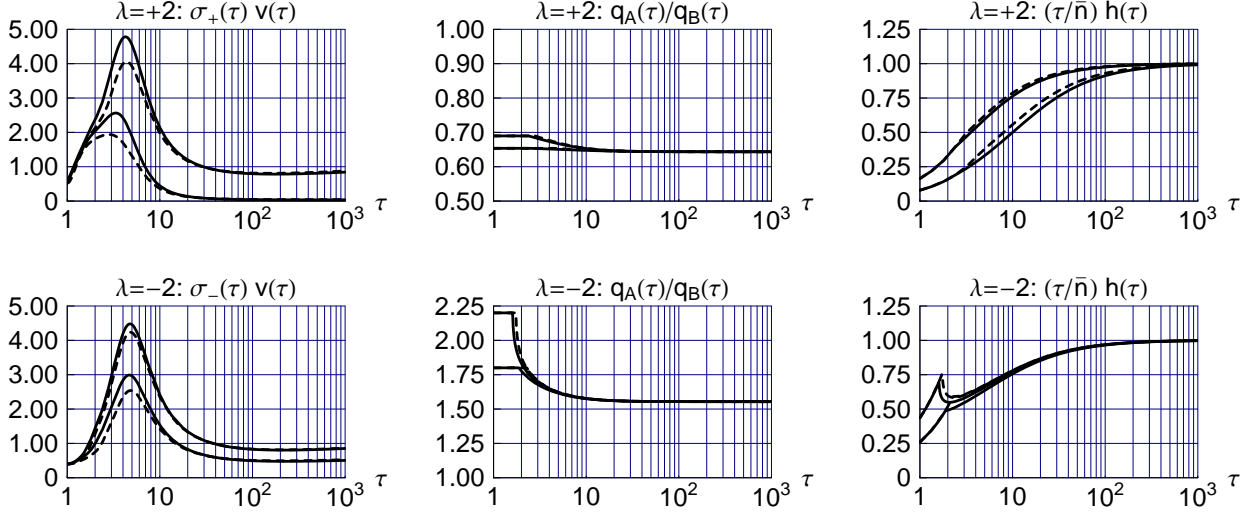


FIG. 1: Numerical solutions of ODEs (3.11) with dimensionless cosmological constant $\lambda = \pm 2$, energy density function (3.10a), and model parameter $\delta = 10^{-10}$. The following auxiliary functions are obtained from $v(\tau)$, $w(\tau)$, and $h(\tau)$: $q_A \equiv \sqrt{\dot{v}^2 + 3h^2 v^2}$ and $q_B \equiv \sqrt{\dot{w}^2 + 3h^2 w^2}$.

Top row ($\lambda = +2$): The boundary conditions are $\{v(1), w(1)\} = \{8/(3/2 - 1/100 + r/25), 8\}$ and $\{\dot{v}(1), \dot{w}(1)\} = \{(3/4 + s/2)/(3/2 - 1/100 + r/25), (3/4 + s/2)\}$ for integers $r = \pm 1$ and $s = \pm 1$. The corresponding values for $h(1)$ follow from (3.12). The dashed lines in the plots refer to the lowest value of $\dot{v}(1)$ coming from $s = -1$. The scaling of the $v(\tau)$ plot uses the function $\sigma_+(\tau) \equiv [1 + 35(\tau - 1)^2]/[10 + 100(\tau - 1)^2 + (\tau - 1)^{2+\bar{p}}]$ and the scaling of the $h(\tau)$ plot uses τ/\bar{n} with exact parameters (\bar{n}, \bar{p}) from (3.15).

Bottom row ($\lambda = -2$): The boundary conditions are $\{v(1), w(1)\} = \{6, 6/(2 + r/5)\}$ and $\{\dot{v}(1), \dot{w}(1)\} = \{(1/2 + s), (1/2 + s)/(2 + r/5)\}$ for $r = \pm 1$ and $s = \pm 1$. The scaling of the $v(\tau)$ plot uses the function $\sigma_-(\tau) \equiv [2 + 12(\tau - 1)^2]/[30 + 120(\tau - 1)^2 + (\tau - 1)^{2+\bar{p}}]$ and the scaling of the $h(\tau)$ plot uses τ/\bar{n} with exact parameters (\bar{n}, \bar{p}) from (3.15).

- (i) With $\bar{p} > 1$ from (3.15), a nonstandard form of q -theory is obtained asymptotically, having growing individual values $q_A(\tau) \sim \tau^{\bar{p}-1}$ and $q_B(\tau) \sim \tau^{\bar{p}-1}$ but a constant ratio q_A/q_B . This behavior allows for both the cancellation of the cosmological constant λ and having $\lim_{\tau \rightarrow \infty} \chi^{-1}(\tau) \equiv \chi_{\text{asympt}}^{-1} = 0$.
- (ii) Even if p were equal to unity [which, with $n = 1$, is also a possible solution of the reduced field equations (3.11)], the present values of X^{-1} and $(\epsilon - \tilde{\epsilon})$ would be negligible for the δ value displayed in (3.13), bringing (3.4a) extremely close to the standard Newtonian result.

- (iii) An entirely open issue is the question of stability (cf. Ref. [6]), where (3.10c) becomes asymptotically $+20 \delta \lambda / (q_A^2 q_B^2)$, which is only positive for the case of $\lambda > 0$ (δ being positive by definition). The possible instability of the $\lambda < 0$ solution may be consistent with the fact that the numerical $\lambda = -2$ solution of Fig. 1 has been found to become ill-behaved for $\delta \sim 10^{-4}$ (i.e., divergent at finite values of τ), whereas the numerical $\lambda = 2$ solution for $\delta \sim 10^{-4}$ remains unchanged compared to the $\delta = 10^{-10}$ case.
- (iv) In the very early universe, i.e., far away from the asymptote, the perturbation equation (3.4a) differs from the standard Einstein expression. This different equation may lead to new effects for the creation and propagation of gravitational waves in the very early universe (assuming the model of this section to be physically relevant). The main focus of the present article is, however, on the Newtonian physics in the final equilibrium state of the universe.

IV. CONCLUSION

The fundamental question addressed in this article is whether or not a vector-field model [3, 4] allows for the dynamic cancellation of an arbitrary cosmological constant Λ without spoiling the local Newtonian gravitational dynamics [5]. The answer found is affirmative, even though the final FRW-type universe obtained ($H \sim 2t^{-1}$) does not quite resemble the actual Universe of our recent past ($H \sim \hat{n}t^{-1}$ for \hat{n} changing from 1/2 to 2/3). The important point is that, as a matter of principle, it is possible to evolve from an initial de-Sitter-type universe [with a cosmological constant $|\Lambda| \sim (E_{\text{Planck}})^4$] to an asymptotic Minkowski spacetime [with $\Lambda_{\text{eff}} \equiv \Lambda + \epsilon(Q_{A0}/Q_{B0}) = 0$ and standard local Newtonian gravitational dynamics].

It is clear that the explicit vector-field example of Sec. III C can be generalized. It may even be possible to appeal to higher-spin fields, perhaps the well-known threeform gauge field (cf. Refs. [6, 9] and further references therein). The most important task, however, is to establish the consistency of this type of vector-field model and to discover the underlying physics.

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NOTE ADDED

The present article considers a particular Λ -cancellation vector-field model which, asymptotically, has the standard Newtonian dynamics on small scales but not an acceptable Hubble expansion on cosmological scales, as noted in the first paragraph of Sec. IV. This article is, in fact, the first of a trilogy of articles.

The second article [13] of the trilogy considers a different model which, asymptotically, gives the standard radiation-dominated FRW universe with $H = (1/2) t^{-1}$ but, most likely, not the standard local Newtonian dynamics.

The third article [14] of the trilogy considers a final model (combining the essential features of the two previous models) which, asymptotically, has both the standard local Newtonian dynamics and the standard radiation-dominated FRW universe. This last article also gives a mathematical discussion of the attractor-type behavior found in the three different vector-field models considered.

A further article [15], a direct follow-up of the present one, discusses the possibility of having an early-universe phase with inflation and a late-universe phase with a dynamically canceled cosmological constant Λ .

Appendix A: Field equations

The action (2.1a) gives the following field equation for the vector field $A_\alpha(x)$:

$$\nabla^\alpha (\zeta \nabla_\alpha A_\beta) = 0, \quad (\text{A1})$$

in terms of the function $\zeta(Q) \equiv \epsilon'(Q)/(2Q)$, where the prime denotes differentiation with respect to Q . For a spatially flat FRW universe, (A1) reduces to

$$\zeta [\partial^\alpha \partial_\alpha + 3H \partial_0 - 3H^2 + \zeta^{-1} \zeta^{,\alpha} \partial_\alpha] A_0 - [2\zeta H \partial^j + H \zeta^{,j}] A_j = 0, \quad (\text{A2a})$$

$$\zeta [\partial^\alpha \partial_\alpha + H \partial_0 - \dot{H} - 3H^2 - \zeta^{-1} \dot{\zeta} H + \zeta^{-1} \zeta^{,\alpha} \partial_\alpha] A_j + [2\zeta H \partial_j + H \zeta_{,j}] A_0 = 0, \quad (\text{A2b})$$

where, in this appendix, an overdot stands for differentiation with respect to the cosmic time t and H is the Hubble parameter defined as \dot{a}/a . Furthermore, the quantity $\zeta^{,\alpha}$ denotes $\partial^\alpha \zeta$, the index α runs from 0 to 3, and the index j runs from 1 to 3.

The energy-momentum tensor $T_{\alpha\beta}(A)$ is obtained by varying the action (2.1a) with respect to the metric $g_{\alpha\beta}$:

$$\begin{aligned} T_{\alpha\beta}(A) &= T_{\beta\alpha}(A) = \epsilon(Q) g_{\alpha\beta} - 2\zeta [A_{\alpha;\gamma} A_{\beta}^{\gamma} + A_{\gamma;\alpha} A_{\beta}^{\gamma}] \\ &+ \nabla^\gamma [\zeta (A_\alpha A_{\gamma;\beta} + A_\beta A_{\gamma;\alpha} + A_\alpha A_{\beta;\gamma} + A_\beta A_{\alpha;\gamma} - A_\gamma A_{\alpha;\beta} - A_\gamma A_{\beta;\alpha})], \end{aligned} \quad (\text{A3})$$

where $A_{\alpha;\gamma}$ denotes the covariant derivative $\nabla_\gamma A_\alpha$ and similarly for other tensors. An alternative form of this energy-momentum tensor is

$$T_{\alpha\beta}(A) = [\epsilon(Q) - \zeta Q^2] g_{\alpha\beta} - 2\zeta T_{\alpha\beta}^{\text{quadratic}}(A) + (\nabla^\gamma \zeta) [A_\alpha A_{\gamma;\beta} + A_\beta A_{\gamma;\alpha} + A_\alpha A_{\beta;\gamma} + A_\beta A_{\alpha;\gamma} - A_\gamma (A_{\alpha;\beta} + A_{\beta;\alpha})], \quad (\text{A4a})$$

$$T_{\alpha\beta}^{\text{quadratic}}(A) = -\frac{1}{2} Q^2 g_{\alpha\beta} + A_{\alpha;\gamma} A_{\beta}^{\gamma} + A_{\gamma;\alpha} A_{\beta}^{\gamma} - \frac{1}{2} \nabla^\gamma [A_\alpha A_{\gamma;\beta} + A_\beta A_{\gamma;\alpha} + A_\alpha A_{\beta;\gamma} + A_\beta A_{\alpha;\gamma} - A_\gamma (A_{\alpha;\beta} + A_{\beta;\alpha})], \quad (\text{A4b})$$

where $T_{\alpha\beta}^{\text{quadratic}}(A)$ agrees with expression (7) of Ref. [4] for $\eta_0 = +1$.

The isotropic *Ansatz* (2.3) reduces (A2a) and (A2b) to a single ODE,

$$\ddot{A}_0 + \left(3H + \dot{\zeta}/\zeta\right) \dot{A}_0 - 3H^2 A_0 = 0, \quad (\text{A5})$$

assuming ζ to be nonzero. Note that ζ in the above equation is a function of A_0 . The implication is that (A5) is, in general, nonlinear in A_0 .

Similarly, we can find the *Ansatz* energy density $\rho(A)$ [from the definition $T_0^0(A) = \rho(A)$] and the isotropic pressure $P(A)$ [from the definition $T_j^i(A) = -P(A) \delta_j^i$]:

$$\rho(A) = \epsilon(Q) - Q \frac{d\epsilon}{dQ}, \quad (\text{A6a})$$

$$P(A) = -\rho(A) + \frac{d}{dt} \left(\frac{H A_0^2}{Q} \frac{d\epsilon}{dQ} \right) - \frac{\dot{A}_0^2}{Q} \frac{d\epsilon}{dQ}, \quad (\text{A6b})$$

with

$$Q^2 \equiv (\dot{A}_0)^2 + 3H^2 A_0^2. \quad (\text{A6c})$$

Finally, the isotropic *Ansatz* (2.3) reduces the Einstein field equations to the following FRW equations:

$$3H^2 = 8\pi G_N [\Lambda + \rho(A)], \quad (\text{A7a})$$

$$2\dot{H} + 3H^2 = 8\pi G_N [\Lambda - P(A)], \quad (\text{A7b})$$

in terms of the vector-field energy density and pressure from (A6).

Appendix B: Quadratic perturbations

Following the discussion of Ref. [5], we consider matter perturbations with timescales and lengths very much smaller than the cosmological timescale $H_0^{-1} \sim 10^{10}$ yr and size

$c/H_0 \sim 10^{26}$ m, defined in terms of the measured Hubble constant $H_0 \sim 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. These matter perturbations are considered to be relevant to the local Newtonian dynamics.

Perturbing around the Dolgov-type solution (3.2), the second-order variation of the Lagrange density (3.1a) of the two vector fields reads:

$$\mathcal{L}^{(2)} = \mathcal{L}_A^{(2)} + \mathcal{L}_B^{(2)} + \mathcal{L}_{AB}^{(2)}, \quad (\text{B1})$$

with

$$\begin{aligned} \mathcal{L}_A^{(2)} = & \frac{1}{2Q_A} \left[\frac{d}{dQ_A} \left(\frac{1}{Q_A} \frac{d\epsilon}{dQ_A} \right) A^{\alpha;\beta} A^{\gamma;\delta} + \frac{d\epsilon}{dQ_A} g^{\alpha\gamma} g^{\beta\delta} \right] \\ & \times \left(\delta A_{\alpha;\beta} \delta A_{\gamma;\delta} - 2 \delta A_{\alpha;\beta} \delta \Gamma_{\gamma\delta}^0 A_0 + \delta \Gamma_{\alpha\beta}^0 \delta \Gamma_{\gamma\delta}^0 A_0^2 \right), \end{aligned} \quad (\text{B2a})$$

$$\begin{aligned} \mathcal{L}_B^{(2)} = & \frac{1}{2Q_B} \left[\frac{d}{dQ_B} \left(\frac{1}{Q_B} \frac{d\epsilon}{dQ_B} \right) B^{\alpha;\beta} B^{\gamma;\delta} + \frac{d\epsilon}{dQ_B} g^{\alpha\gamma} g^{\beta\delta} \right] \\ & \times \left(\delta B_{\alpha;\beta} \delta B_{\gamma;\delta} - 2 \delta B_{\alpha;\beta} \delta \Gamma_{\gamma\delta}^0 B_0 + \delta \Gamma_{\alpha\beta}^0 \delta \Gamma_{\gamma\delta}^0 B_0^2 \right), \end{aligned} \quad (\text{B2b})$$

$$\begin{aligned} \mathcal{L}_{AB}^{(2)} = & \frac{1}{Q_A Q_B} \left[\frac{d^2 \epsilon}{dQ_A dQ_B} A^{\alpha;\beta} B^{\gamma;\delta} \right] \\ & \times \left(\delta A_{\alpha;\beta} \delta B_{\gamma;\delta} - \delta A_{\alpha;\beta} \delta \Gamma_{\delta\gamma}^0 B_0 - \delta B_{\gamma;\delta} \delta \Gamma_{\alpha\beta}^0 A_0 + \delta \Gamma_{\alpha\beta}^0 \delta \Gamma_{\gamma\delta}^0 A_0 B_0 \right), \end{aligned} \quad (\text{B2c})$$

where $\delta A_\alpha(x)$ and $\delta B_\alpha(x)$ are the vector perturbations and $\delta \Gamma_{\alpha\beta}^0(x) \equiv (1/2) [h_{0\alpha,\beta}(x) + h_{0\beta,\alpha}(x) - h_{\alpha\beta,0}(x)]$ contains the metric perturbation $h_{\alpha\beta}(x)$.

Quadratic terms of order $H h \partial h$ and $H^2 h^2$ have been calculated but are not given explicitly in (B2), because they are subleading compared to the $(\partial h)^2$ terms shown [the relevant timescales for the Newtonian dynamics (e.g., in the solar system) are very much smaller than the cosmological timescale H^{-1}]. Remark, finally, that the perturbation fields δA_α , δB_α , and $h_{\alpha\beta}$ entering (B2) are denoted \hat{v}_α , \hat{w}_α , and $\hat{h}_{\alpha\beta}$ in Sec. IIIB; see also the earlier definitions in (2.5).

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